

Vanishing Signatures, Orbit Closure, and the Converse of the Holant Theorem

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Motivating example: trace indistinguishability

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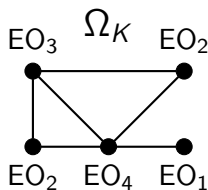
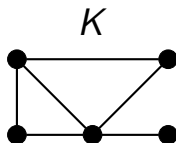
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- Goal: generalize to higher-arity tensors.

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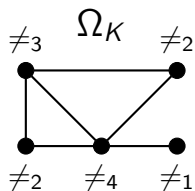
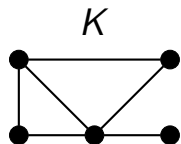
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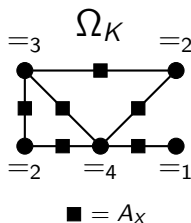
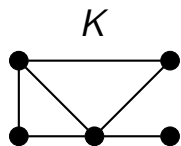
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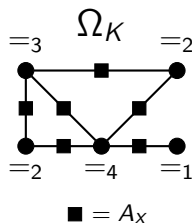
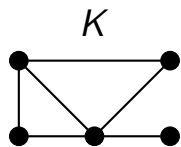
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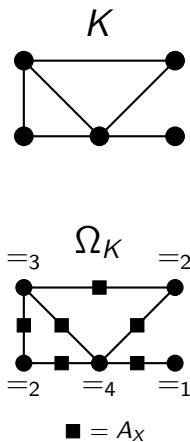
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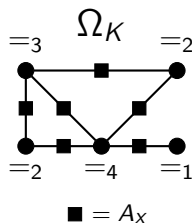
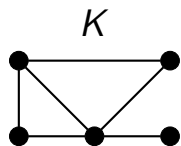
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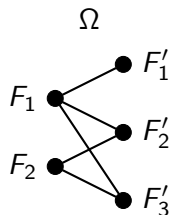


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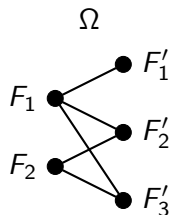
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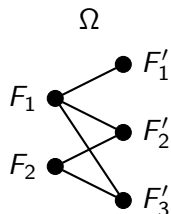
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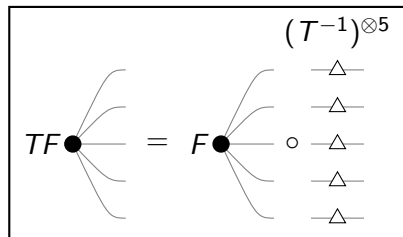
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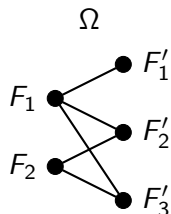


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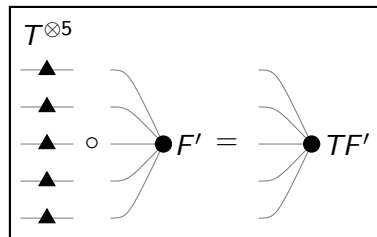
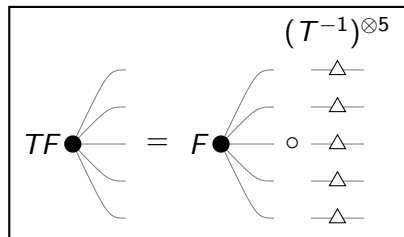


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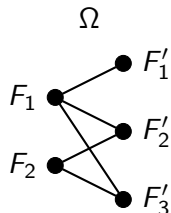


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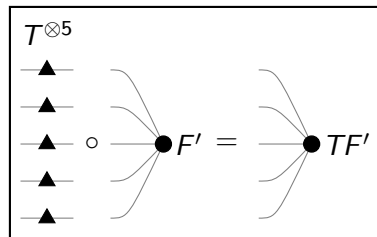
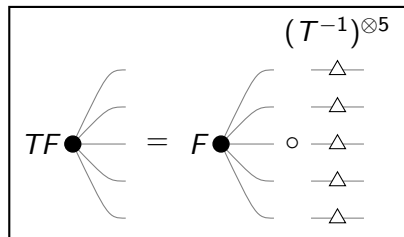


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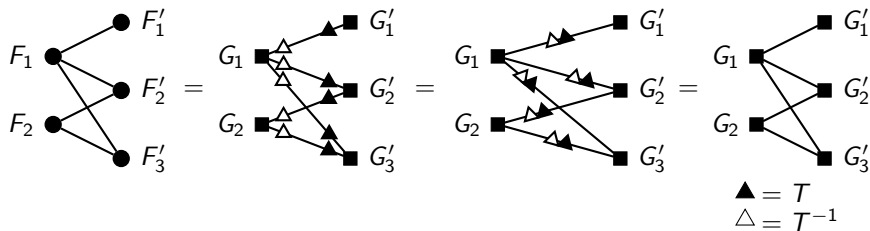
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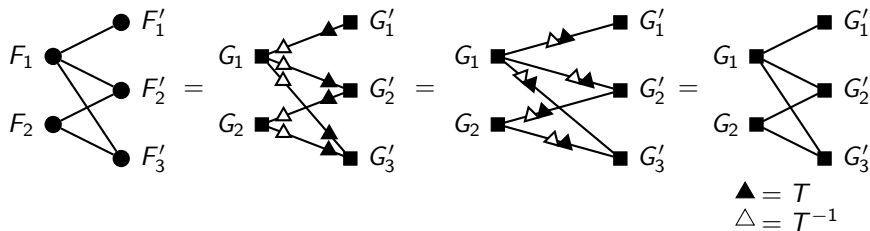
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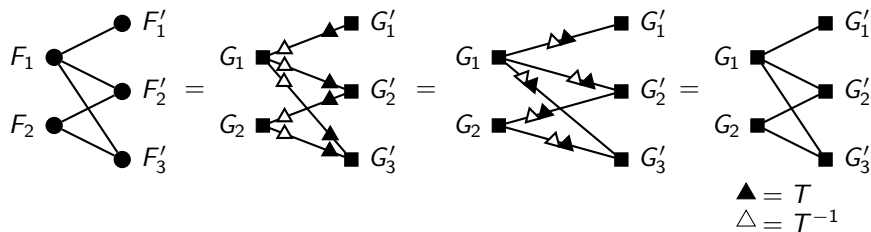
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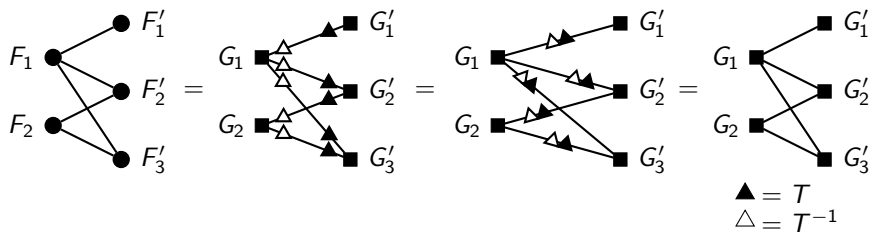
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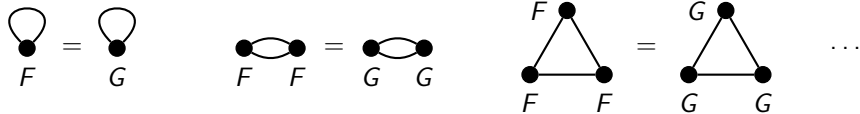
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- We prove two near-converses.

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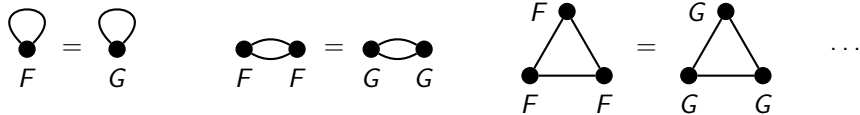
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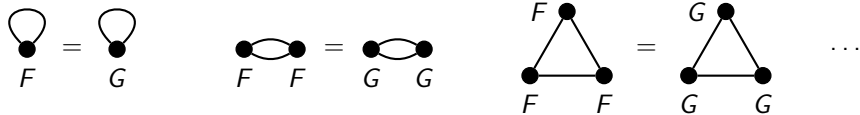


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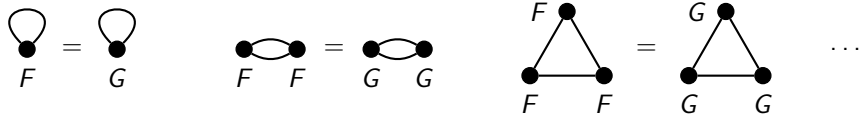
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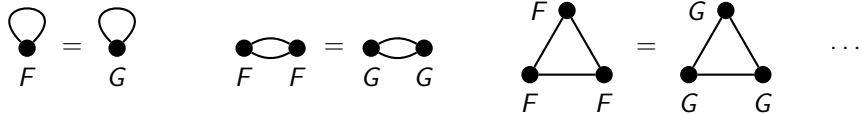
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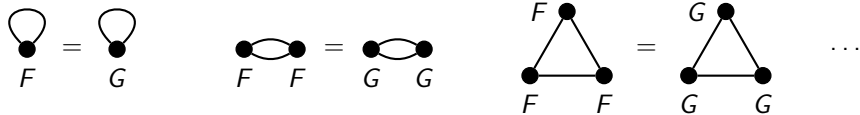
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- Also previous connections with tensor network theory [AMN⁺23].

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- But characterization was open for **graphs of degree $\leq d$** .

Homomorphism Indistinguishability

- Graphs X and Y are **homomorphism-indistinguishable** over \mathcal{G} if $\#\text{hom}(K, X) = \#\text{hom}(K, Y)$ for every $K \in \mathcal{G}$.

Theorem (Lovász [Lov67])

X and Y are isomorphic iff X and Y are homomorphism-indistinguishable over *all graphs*.

- Get relaxations of isomorphism for other \mathcal{G}
 - e.g. *trees, cycles, planar graphs, bounded treewidth*, etc.
- But characterization was open for *graphs of degree $\leq d$* .

Corollary

X and Y are homomorphism-indistinguishable over *graphs of degree $\leq d$* iff the GL_q -orbit closures of $A_X|\{=n\}_{n \leq d}$ and $A_Y|\{=n\}_{n \leq d}$ intersect.

- $\mathcal{F} | \mathcal{F}'$ is **quantum-nonvanishing** if the contraction bilinear form is nondegenerate.

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Theorem (The Conditional Converse)

If $\mathcal{F} | \mathcal{F}'$ and $\mathcal{G} | \mathcal{G}'$ are Holant-indistinguishable & quantum-nonvanishing, then there is a $T \in GL_q$ such that $\mathcal{F} | \mathcal{F}' = T(\mathcal{G} | \mathcal{G}')$.

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Theorem (The Conditional Converse)

If $\mathcal{F} | \mathcal{F}'$ and $\mathcal{G} | \mathcal{G}'$ are Holant-indistinguishable & *quantum-nonvanishing*, then there is a $T \in GL_q$ such that $\mathcal{F} | \mathcal{F}' = T(\mathcal{G} | \mathcal{G}')$.

Corollary

If A_X, A_Y are *invertible*, then $X \cong Y$ iff X and Y are homomorphism-indistinguishable over *graphs of degree ≤ 3* .

Thank you!

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